"Single reference" Bloch eq.

- A transcript of Sch eq., which is
  - energy independent
  - perturbative.

$\hat{H}\Psi = E\Psi$ sol

$\hat{P}^2 = P$, $\hat{O}^2 = O$, $\hat{P} + \hat{O} = 1$ $(\Rightarrow \hat{O}\hat{P} = \emptyset)$

$H = H^0 + V$ partition

$H^0 \phi_a^0 = E_a^0 \phi_a^0$ 0^{th} sol

! $\hat{O} = |\phi_0^0\rangle\langle\phi_0^0|$ 1-D projector

$\hat{P} = 1 - \hat{O} = \sum_{a \neq 0} |\phi_a^0\rangle\langle\phi_a^0|$

! $\hat{\Omega} = |\Psi\rangle\langle\phi_0^0|$

$(|\Psi\rangle = \hat{\Omega}|\phi_0^0\rangle)$

We already have seen: ②

$$\hat{H}\hat{\Omega} = \hat{\Omega}\hat{H}\hat{\Omega} \qquad \text{if } \hat{\Omega} = |\Psi\rangle\langle\phi^0|$$

PT:

$$\hat{H}^0\hat{\Omega} + \hat{V}\hat{\Omega} = \hat{\Omega}\hat{H}^0\hat{\Omega} + \hat{\Omega}\hat{V}\hat{\Omega}$$

$$\hat{H}^0\Omega - \underbrace{\hat{\Omega}\hat{H}^0\hat{\Omega}}_{?} = \hat{\Omega}\hat{V}\hat{\Omega} - \hat{V}\hat{\Omega}$$

$$\hat{\Omega}\hat{H}^0\hat{\Omega} = |\Psi\rangle\underbrace{\langle\phi^0|\hat{H}^0}_{E^0\langle\phi^0|}|\Psi\rangle\langle\phi^0|$$

$$= |\Psi\rangle\underbrace{\langle\phi^0|\Psi\rangle}_{1}\underbrace{\langle\phi^0|E^0}_{\langle\phi^0|\hat{H}^0}$$

$$= \underbrace{|\Psi\rangle\langle\phi^0|}_{\Omega}\hat{H}^0 = \hat{\Omega}\hat{H}^0$$

$$\hat{H}^0\hat{\Omega} - \hat{\Omega}\hat{H}^0 = \hat{\Omega}\hat{V}\hat{\Omega} - \hat{V}\hat{\Omega}$$

$$\boxed{\begin{aligned}[\hat{\Omega}, \hat{H}^0] &= \hat{V}\hat{\Omega} - \hat{\Omega}\hat{V}\hat{\Omega} \\ &= (1 - \hat{\Omega})\hat{V}\hat{\Omega}\end{aligned}}$$

(Generalized) Bloch eq.