# Exactly-solvable position-dependent mass Schrödinger equation for the Thomas-Fermi and Harmonic Oscillator potentials.

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Abstract

The aim of this work, is to obtain the exactly-solvable position-dependent mass Schrödinger equation (PDMSE) for the Thomas-Fermi and Harmonic Oscillator potentials. To attain that purpose, the PDMSE in the variable *x* is transformed into a standard Schrödinger-like equation with constant mass (CMSE), in a new the variable *u*, by means of a point canonical transformation scheme. This approach applied to the PDMSE, allow us to obtain the transformation that should be used to find the exactly solvable CMSE. In that case, the former potential associated to the PDMSE and the potential involved in the CMSE are related through a Riccati-type relationship that includes the equivalent of the Witten superpotential that determines the position-dependent mass distribution *m(x)* leading to exactly-solvable PDMSE. As a useful application of the proposal, we assume that the Witten superpotential is constant to find *m(x)* and consequently the exactly-solvable *V(x)* Thomas-Fermi and Harmonic Oscillator potentials. In that case, it is considered an exponential-type *V(u)* potential leading to Bessel-type eigenfunctions for the Thomas-Fermi and to Laguerre-type solutions for harmonic oscillator potentials for the PDMSE, as particular cases. Beyond the worked example, the approach is general and can be useful in the study of the electronic properties of materials in which the carrier effective mass depends on the position as well as its application in the search of new potentials suitable for the study of quantum systems.

*Keywords* : Position-dependent mass, Point canonical transformation, Thomas-Fermi potential, Schrödinger equation.